VISCOUS DISSIPATION EFFECTS ON MHD FLOW PAST A PARABOLIC STARTED VERTICAL PLATE WITH VARIABLE TEMPERATURE AND MASS DIFFUSSION

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Abstract— Viscous dissipation effects on the unsteady mhd free convective flow past a parabolic starting motion of the infinite vertical plate with variable temperature and variable mass diffusion is investigated. The plate temperature and the concentration level near the plate are raised linearly with time. The dimensionless governing unsteady, non-linear, coupled partial equations are solved by using the unconditionally stable explicit finite difference method of DuFort – Frankel's type. The effect of velocity profiles are studied for different physical parameters like Eckert number, Prandtl number, thermal Grashof number, mass Grashof number, Schmidt number, magnetic parameter and time. It is observed that the velocity increases as the value of the thermal Grashof number or mass Grashof number increase. The trend is just reversed with respect to the Schmidt number.

Index Terms— Viscous dissipation, MHD, parabolic started vertical plate, variable temperature, variable Concentration,.etc.

1 INTRODUCTION

There are many applications for the parabolic motion such as solar cookers, solar concentrators and parabolic trough solar collector. A parabolic concentrator type solar cooker has a wide range of applications like baking, roasting and distillation due to its unique property of producing a practically higher temperature of nearly 250^{0}C and hence it provides inconvenience to the user due to high amount of glare. Solar concentrators have their applications in increasing the rate of evaporation of waste water, in food processing, for making drinking water from brackish and sea water. It produces a high temperature around 250^{0}C and the food gets cooked in less time [1].

A parabolic trough solar collector system will provide within next decade a significant contribution to efficient, economical, sustainable renewable and clean energy supply with positive effect on environmental activities and it is designed to concentrate sun rays via parabolic curved solar reflectors onto a heat absorber element – a receives – located in the optical focal line of the collectors [2].

Muthucumaraswamy and Geetha [3] studied the effects of parabolic motion on an isothermal vertical plate with constant mass flux. Muthucumaraswamy and Neel Amstrong [4] examined the mass transfer effects on flow past a parabolic started vertical plate with variable temperature and mass diffusion. Visalakshi and Vasanthabhavam [5] investigated the heat and mass transfer effects on flow past parabolic started vertical plate with constant heat flux. Muralidharan and Muthucumaraswamy [6] discussed the parabolic started flow past an infinite vertical plate with uniform heat flux and variable mass diffusion.

The study of simultaneous heat and mass transfer in the presence of magneto hydrodynamics plays an important role in petroleum industries, geophysics and in astrophysics. It also finds applications in many engineering problems such as magneto hydrodynamics generator, plasma studies, in the study of geological formations, in exploration and thermal recovery of oil, and in the assessment of aquifers, geothermal reservoirs and underground nuclear waste storage sites. MHD flow has applications in metrology, solar physics and in the movement of earth's core.

Heat transfer effects on impulsively started an infinite vertical plate in the presence of magnetic field was studied by Soundalgekar et al. [7]. Agarwal et. el [8] studied free convection due to thermal and mass diffusion in laminar flow of an accelerated infinite plate in the presence of magnetic field. Agarwal et al. [9[further extended the problem of unsteady free convective flow and mass diffusion of an electrically conducting elasto-viscous fluid past a parabolic starting motion of the infinite vertical plate with transverse magnetic field. Muthucumaraswamy and Velmurugan [10] examined the hydromagnetic flow past a parabolic started vertical plate in the presence of homogeneous chemical reaction of first order.

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In all the investigations mentioned above, viscous mechanical dissipation is neglected. Viscous mechanical dissipation effects are very important because of its varied applications in the field of cosmical and geophysical flows. Grebhart and Mollendorf [11] considered the viscous dissipation in external natural convection flows. Soundalgekar [12] studied the viscous dissipation effect on unsteady free convection flow past an infinite, vertical porous plate with constant suction. Later the same problem with variable suction was investigated by Soundalgekar [13]. Mahajan et al. [14] reported the influence of viscous heating dissipation effects in natural convective flows, showing that the heat transfer rates are reduced by an increase in the dissipation parameter.

Manohar and Nagarajan [15] discussed the mass transfer effects on free convective flow of an incompressible viscous dissipative fluid. Loganathan and Sivapoornapriya [16] studied the viscous dissipation effects on unsteady natural convective flow past an infinite vertical plate with uniform heat and mass flux. Hemanth Poonia and Chaudhary [17] analyzed the MHD free convection and mass transfer flow over an infinite vertical porous plate with viscous dissipation.

Motivated by the above mentioned investigations and applications, an attempt is made to study an unsteady flow of a viscous incompressible fluid past an infinite vertical plate with variable temperature and variable diffusion under the influence of magnetic field and viscous dissipation. Numerical solution has been obtained for velocity, temperature and concentration by using unconditionally stable method of DuFort-Fronkel.

2 FORMULATION OF THE PROBLEM

Consider the unsteady flow of a viscous incompressible fluid past an infinite vertical plate with variable temperature and variable diffusion under the influence of viscous dissipation. The x' - axis is taken along the plate in the vertically upward direction and the y' - axis is taken normal to the plate. At time $t' \le 0,$ the plate and fluid are at the same temperature T'_∞ and concentration C'_∞. At $t' > 0$, the plate is started with a velocity $u = u_0 t'^2$ in its own plane against gravitational field. The plate temperature is raised uniformly and the mass is diffused from the plate to the fluid is made to raise linearly with time t' also subjected to a uniform magnetic field of strength B_0 is assumed to be applied normal to the plate. Since the plate is infinite in length, all the terms in the governing equations will be independent of x' and there is no flow along y' - direction. Then under usual Boussinesq's approximation for unsteady parabolic starting motion is governed by the following equations.

$$\frac{\partial u'}{\partial t'} = \nu \frac{\partial^2 u'}{\partial y'^2} + g\beta\left(T' - T'_\infty\right) + g\beta^*\left(C' - C'_\infty\right) - \frac{\sigma B_0^2}{\rho} u' \quad (1)$$

$$\rho C_p \frac{\partial T'}{\partial t'} = \kappa \frac{\partial^2 T'}{\partial y'^2} + \mu \left(\frac{\partial u'}{\partial y'}\right)^2 \quad (2)$$

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y'^2} \quad (3)$$

With the following initial and boundary conditions:

$$\left.\begin{array}{ll} t' \le 0: u' = 0, & T' = T'_\infty, \\ & C' = C'_\infty \qquad \text{for all } y' \\ t' > 0: u' = u_0 t'^2, & T' = T'_\infty + \left(T'_w - T'_\infty\right) A t', \\ & C' = C'_\infty + \left(C'_w - C'_\infty\right) A t' \text{ at } y' = 0 \\ \quad u' \to 0, & T' \to T'_\infty, \\ & C' \to C'_\infty \qquad \text{as } y' \to \infty \end{array}\right\} \quad (4)$$

On introducing the following non-dimensional quantities:

$$\left.\begin{array}{l} U = u'\left(\frac{u_0}{\nu^2}\right)^{\frac{1}{3}},\ t = t'\left(\frac{u_0^2}{\nu}\right)^{\frac{1}{3}},\ Y = y'\left(\frac{u_0}{\nu^2}\right)^{\frac{1}{3}}, \\ Gr = \frac{g\beta\left(T' - T'_\infty\right)}{\left(\nu u_0\right)^{\frac{1}{3}}},\ ,\ \Pr = \frac{\mu C_p}{\kappa},\ \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty}, \\ C = \frac{C' - C'_\infty}{C'_w - C'_\infty}, Gc = \frac{g\beta\left(C' - C'_\infty\right)}{\left(\nu u_0\right)^{\frac{1}{3}}},\ Sc = \frac{\nu}{D}, \\ Ec = \frac{\left(\frac{u_0}{\nu}\right)^{-\frac{1}{3}}}{C_p\left(T'_w - T'_\infty\right)},\ M = \frac{\sigma B_0^2}{\rho}\left(\frac{\nu}{u_0^2}\right)^{\frac{1}{3}} \end{array}\right\} \quad (5)$$

With the help of (5), equations (1), (2), and (3) reduces to

$$\frac{\partial U}{\partial t} = \frac{\partial^2 U}{\partial Y^2} + Gr\theta + GcC - MU \quad (6)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial Y^2} + Ec\left(\frac{\partial u}{\partial Y}\right)^2 \quad (7)$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial Y^2} \quad (8)$$

The initial and boundary conditions in non-dimensional form are

$$\left.\begin{array}{ll} t \le 0: & U = 0,\quad \theta = 0,\ C = 0 \quad \text{for all } \ Y \\ t > 0: & U = t^2,\ \theta = t,\ C = t \quad \text{at } \ Y = 0 \\ & U \to 0,\ \theta \to 0,\ C \to 0 \text{ as } \ Y \to \infty \end{array}\right\} \quad (9)$$

3 METHOD OF SOLUTION

Equations (6) – (8) are coupled non-linear partial differential equations and are to be solved under the initial and boundary conditions of equations (9). However exact or approximate solutions are not possible for this set of equations and hence we solve these equations by the unconditionally stable explicit finite difference method of DuFort – Frankel's type as explained by Jain et. al. [14]. The finite difference equations corresponding to equations (6) – (8) are as follows:

$$\left(\frac{U_{i,j+1}-U_{i,j-1}}{2\Delta t}\right)=\left(\frac{U_{i-1,j}-U_{i,j+1}-U_{i,j-1}+U_{i+1,j}}{(\Delta Y)^2}\right)+\frac{Gr}{2}\left(\theta_{i,j+1}+\theta_{i,j-1}\right)+\frac{Gc}{2}\left(C_{i,j+1}+C_{i,j-1}\right)-M\left(\frac{U_{i,j+1}+U_{i,j-1}}{2}\right) \tag{10}$$

$$\left(\frac{\theta_{i,j+1}-\theta_{i,j-1}}{2\Delta t}\right)=\frac{1}{\Pr}\left(\frac{\theta_{i-1,j}-\theta_{i,j+1}-\theta_{i,j-1}+\theta_{i+1,j}}{(\Delta Y)^2}\right)+Ec\left(\frac{U_{i+1,j}-U_{i,j}}{\Delta Y}\right)^2 \tag{11}$$

$$\left(\frac{C_{i,j+1}-C_{i,j-1}}{2\Delta t}\right)=\frac{1}{Sc}\left(\frac{C_{i-1,j}-C_{i,j+1}-C_{i,j-1}+C_{i+1,j}}{(\Delta Y)^2}\right) \tag{12}$$

Initial and boundary conditions take the following forms

$$\left.\begin{array}{lll} U_{i,0}=0 & \theta_{i,0}=0, & C_{i,0}=0 \quad for\ \ all\ \ i \\ U_{0,j}=(j\Delta t)^2, & \theta_{0,j}=j\Delta t, & C_{0,j}=j\Delta t \\ U_{L,j}=0, & \theta_{L,j}=0, & C_{L,j}=0 \end{array}\right\} \tag{13}$$

Where L corresponds to ∞, the suffix $'i'$ corresponds to Y and $'j'$ corresponds to t.

Also $\Delta t = t_{j+1}-t_j$ and $\Delta Y = Y_{i+1}-Y_i$.

4 RESULTS AND DISCUSSIONS

In order to point out the effects of various parameters on flow characteristic, the following discussion is set out. The values of the Prandtl number are chosen air (Pr = 0.71) and the Schmidt number are chosen water vapour (0.60). The numerical values of the velocity, temperature and concentration are computed for the different physical parameters like Magnetic parameter (M = 1), Prandtl number (Pr = 0.71), thermal Grashof number (Gr = 2), mass Grashof number (Gc = 5), Schmidt number (Sc = 0.60), Eckert number (Ec = 1) and time (t = 0.4).

In fig. 1, velocity profile for various values of Eckert number is plotted. Eckert number expresses the relation between kinetic energy and the enthalphy. It is used to characterize the dissipative process. It is evident that viscous dissipative heat raises the velocity of the fluid. Hence, the velocity increases with increasing values of Eckert number.

In fig. 2, velocity profile for various values of Prandtl number is shown. Prandtl number being the ratio of momentum diffusivity to thermal diffusivity will influence the fluid flow as long as the velocity field and the temperature field are coupled. Hear the velocity decreases with increasing values of Prandtl number.

Fig. 3 depicts the velocity profile for different values of thermal Grashof number (Gr = 2, 5) and mass Grashof number (Gc = 5, 10) are observed. It is noticed that the velocity increases with the thermal Grashof number or mass Grashof number. The effect of thermal Grashof number is very dominant, since the temperature of the plate is assumed to be uniform.

Fig. 4 illustrates the velocity profile for different values of Schmidt number. As the graph represents the velocity increases as the Schmidt number decreases. The relative variation with regard to the velocity can be noticed.

Fig. 5 revels that magnetic parameter on fluid velocity and we observed that an increase in magnetic parameter M the velocity decreases. It is due to face that the application of transverse magnetic field will result a resistive type force (Lorentz force) similar to drag force, which tends to resist the fluid flow and thus reducing its velocity.

The effect of time *'t'* on the velocity is shown in Fig. 6. It is obvious from the figure that the velocity increases with the increase of time *'t'*.

The effect of Eckert number on the temperature is shown in fig 7. The effect of viscous dissipation on flow field is to increase the energy, yielding a greater fluid temperature and as a consequence greater buoyancy force. The increase in the buoyancy force due to an increase in the dissipation parameter enhances the temperature.

The effect of Prandtl number plays an important role in temperature field it is depict in fig. 8. It was observed that the temperature increases with decreasing Prandtl number. This shows that the heat transfer is more in air than in water.

Fig. 9 represents the effect of concentration profiles for different Schmidt number. The effect of concentration is important in concentration field. The profiles have the common feature that the concentration decreases in a monotone fashion from the surface to a zero value far away in the free stream. It is observed that the wall concentration increases with decreasing values of the Schmidt number.

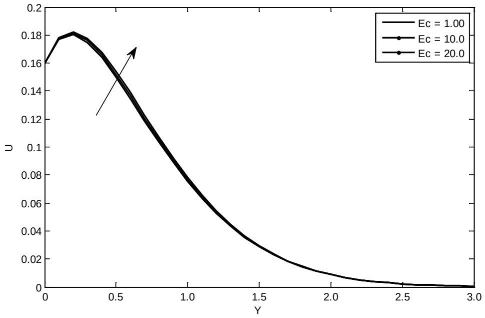

Fig. 1: Velocity profiles for different values of Eckert num-

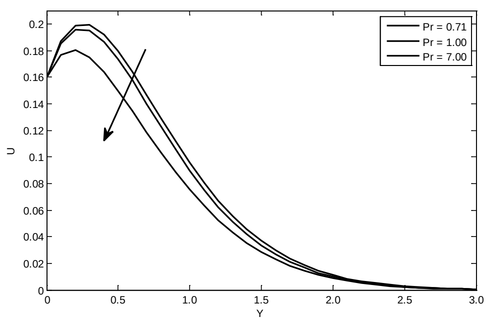

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Fig. 2: Velocity profiles for different values of Prandtl number

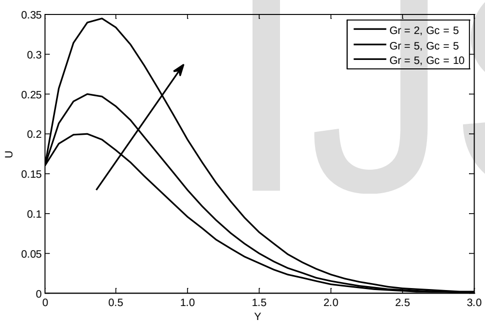

Fig. 3: Velocity profiles for different values of thermal Grashof number and mass Grashof number

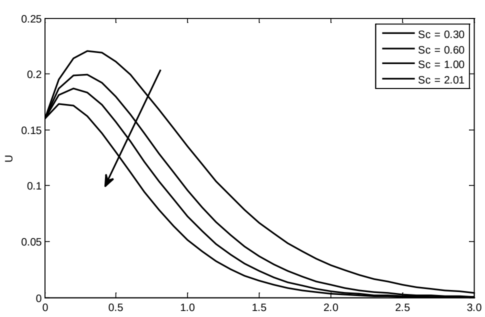

Fig. 4: Velocity profiles for different values of Schmidt number

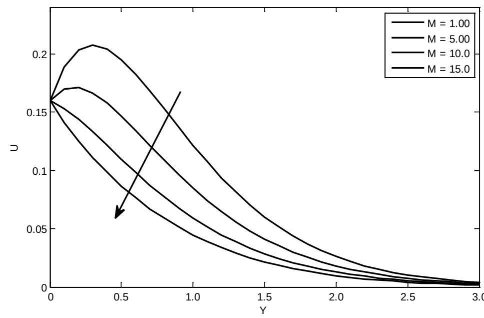

Fig. 5: Velocity profiles for different values of Magnetic parameter

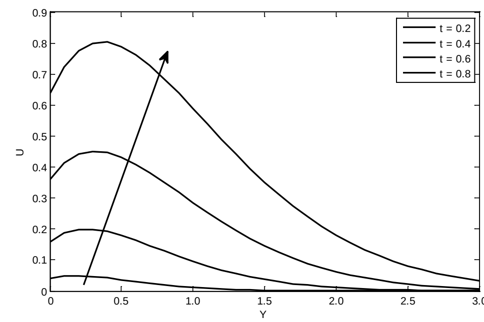

Fig. 6: Velocity profiles for different time levels

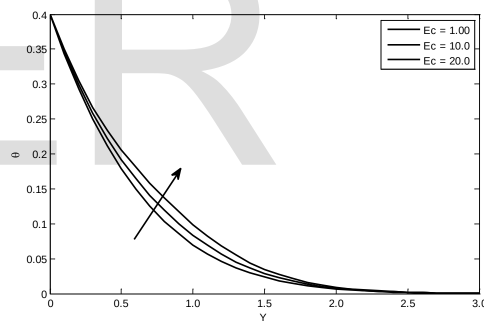

Fig. 7: Temperature profiles for different Eckert number

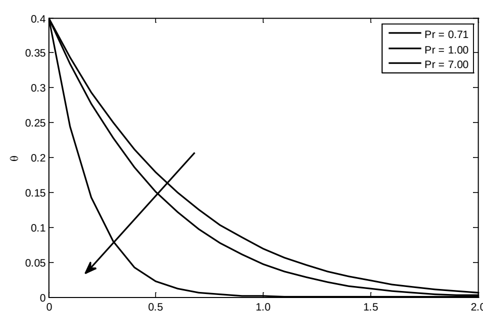

Fig. 8: Temperature profiles for different Prandtl number

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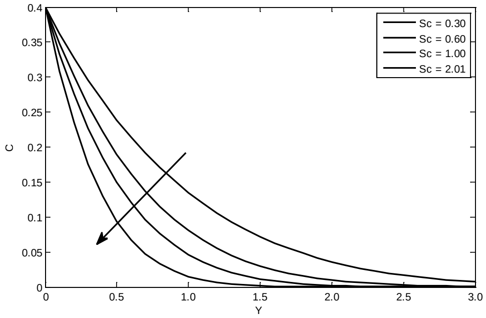

Fig. 9: Concentration profiles for different Schmidt number

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6 CONCLUSIONS

The present investigation brings out the following interesting features of physical interest on the flow velocity, temperature and concentration:

- Velocity increases with increase in the Eckert number (Ec)
- Velocity decreases with increase Prandtl number (Pr)
- Velocity increases with increase in the thermal Grashof number (Gr) or mass Grashof number (Gc).
- Velocity increases with decrease Schmidt number (Sc)
- Velocity decreases with increase Magnetic parameter (M)
- Velocity increases with increasing values of the time (t)

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- Temperature increases with decrease in Prandtl number (Pr)
- Temperature increases with increase in Eckert number (E_{c})
- Concentration decreases with increase in (Sc)